



The Mission and Requirements of a Turbulence Model

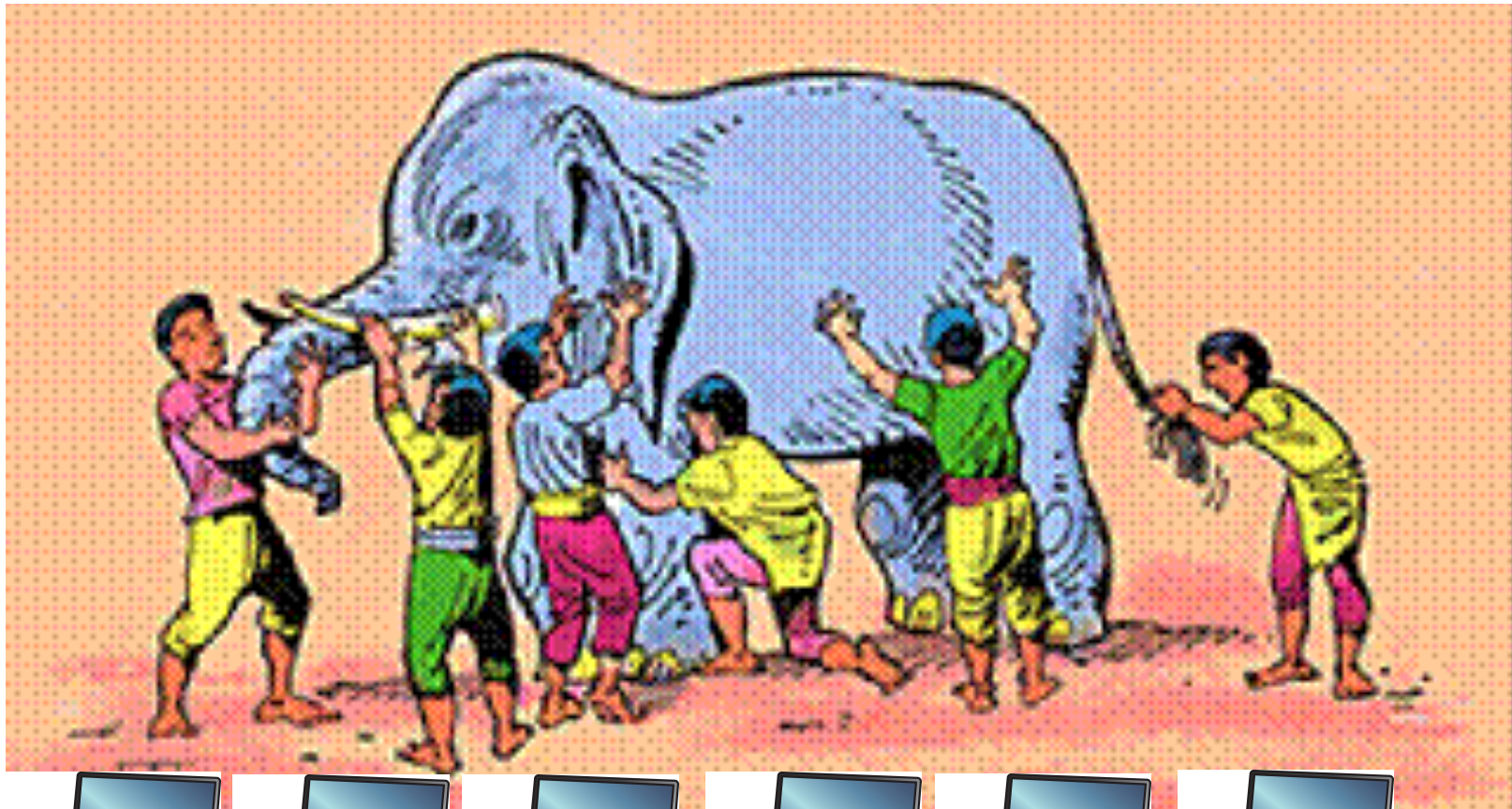
Philippe Spalart
Boeing Commercial Airplanes

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Outline

- Background
 - Classical (RANS) modeling versus turbulence-resolving methods (LES+) in the 21st century
 - Stagnation in RANS modeling?
 - Impetus for Machine Learning
- Mission of a turbulence model
 - Universality
 - Numerical well-posedness
 - Physical justification
 - Documentation and version control
- Constraints on a turbulence model
 - Hard
 - Intermediate
 - Soft
- The way forward
 - Effective strategies, based on both Natural and Artificial Intelligence
 - Accurate, relevant data from DNS and experiments
 - Critical examination of machine-learning studies

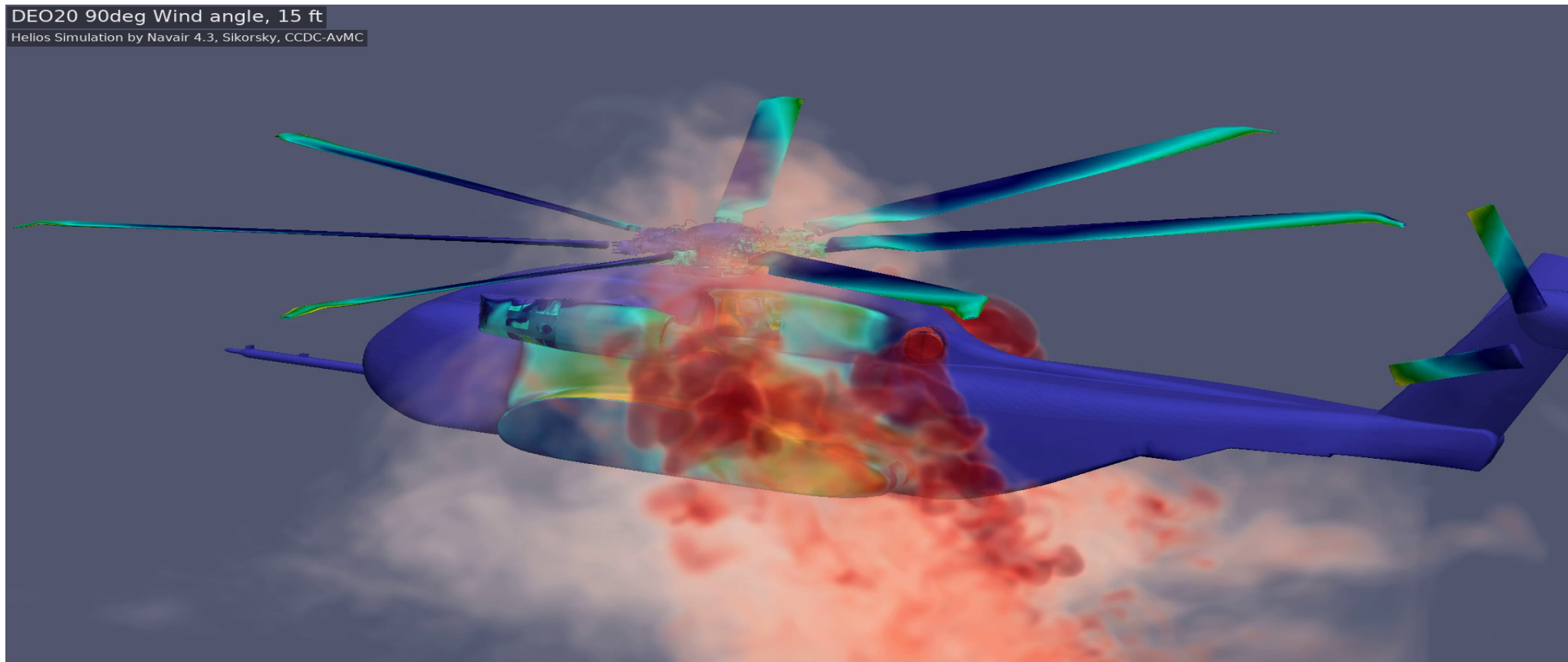
The Turbulence Community?



Background

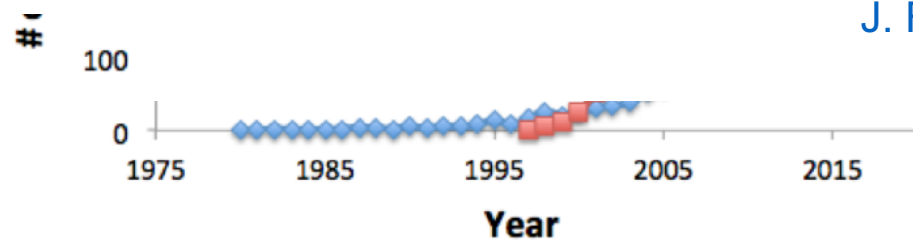
- Turbulence models are extremely useful
 - Approaches having NO empirical content may well not work within the 21st century, if ever
- This will be the era of hybrid RANS-LES methods
 - Pure LES, even Wall-Modeled, is impractical for high-Reynolds-number attached flows
 - $\delta \sim 1.2\text{mm}$ for the boundary layer at the leading edge of a 787! (even if assumed turbulent)
 - Pure RANS, even Unsteady RANS, not accurate enough for complex or separated flows
- Perception of stagnation in RANS modeling
 - Core of SA and SST models appeared in 1992
 - Improvements such as SARC and QCR are not insignificant
 - Failure of Reynolds-Stress Models to deliver “automatic” accuracy advantage
 - Modeling is a “strange” field, hard to teach
 - It combines shameless empiricism and non-trivial math
- Widespread calls for Machine Learning!
 - Success of ML in other fields
 - Availability of Big Data
 - Papers accepted in the best journals
 - Powerful HiFiTURB European research program, and others
- Well over half of ML studies produced “things” that are NOT turbulence models
 - “In My Humble Opinion” (I have been praised for “brutal honesty” before)
 - They violate constraints that should be obvious

DEO20 90deg Wind angle, 15 ft
Helios Simulation by Navair 4.3, Sikorsky, CCDC-AvMC



5 20 100 500 2000 10000 50000 200000 1E+06

J. Forsythe, CREATE



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Mission of a Turbulence Model

- Reminder of the usual structure of the equations
- Universality
 - Menter's "highly adjustable" GEKO model
- Numerical well-posedness
- Physical justification
- Documentation and version control
- Compatibility with LES, in hybrid settings

Usual Structure of a Turbulence Model

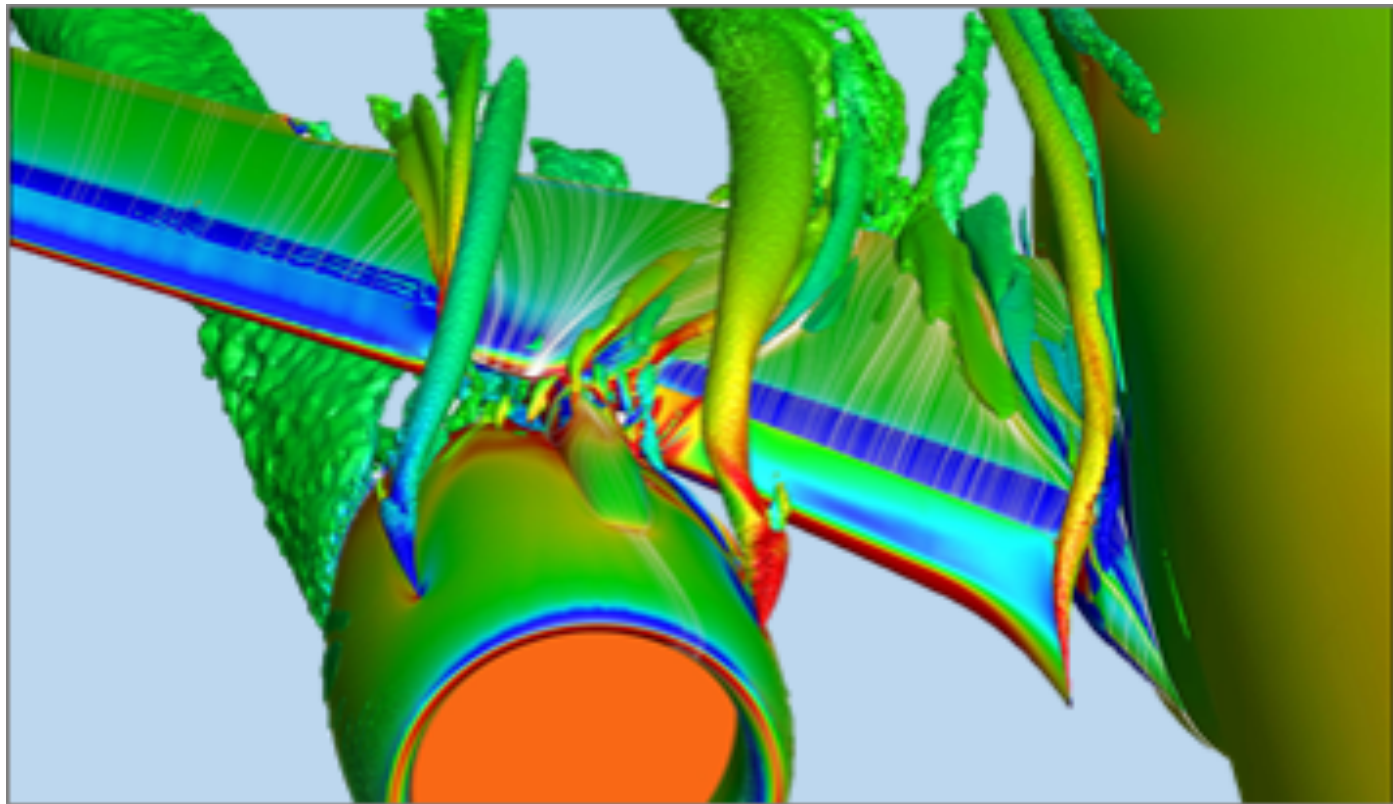
- It mimics the structure of the exact transport equations
- For any quantity ϕ ,

$$\frac{D\phi}{Dt} = \textit{Production} - \textit{Destruction} + \textit{Diffusion} + \textit{Corrections}$$

- In *some* models, *some* terms are exact
- Source terms P and De use local turbulence quantities and velocity derivatives
 - Destruction may use the wall distance d, which does NOT appear in exact equations
- Diffusion Di uses up to second derivatives of ϕ
- Eddy-viscosity constitutive relation is simply dimensional analysis
 - If it is non-linear, largely arbitrary combinations are used
- Features are up to the creator of the model!
 - E.g., QCR and Lag model use DS_{ij}/Dt
- This “freedom” is an opportunity and a danger
 - Especially in the ML era: more features can be tried

Universality?

- “A model” is applied to numerous different flow modules in a solution
- Industrial practice is not zonal (meaning, set by user)



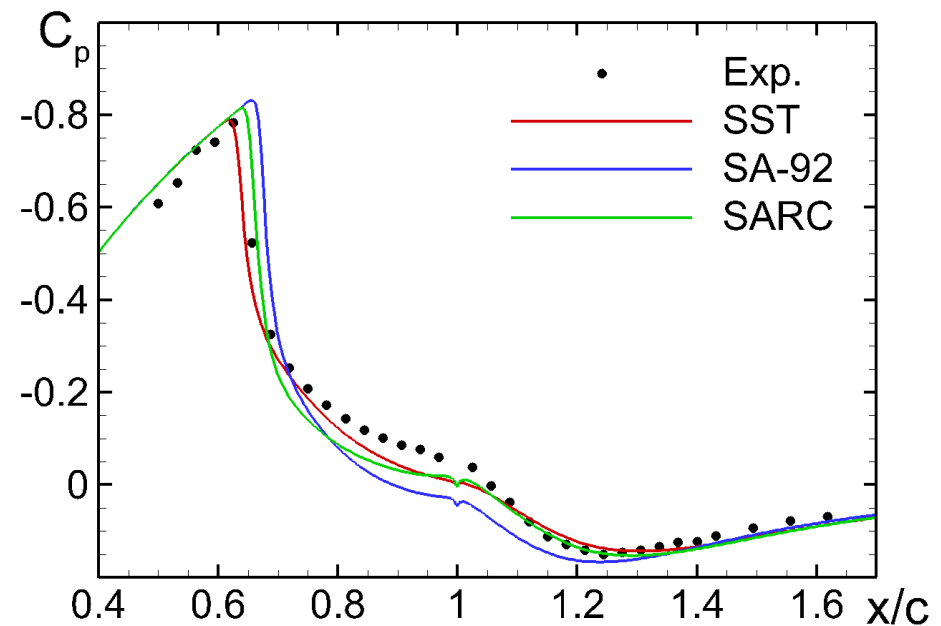
Courtesy J. Slotnick and Airbus! Wall pressure, and field vorticity

Traditional Competition Between Models

- The Bachalo-Johnson flow has been a major test case
 - Representative of airliner wings in cruise
 - Shock-Induced Separation
- SST has better shock position than SA
 - But SARC is very competitive
- The models were static for many years

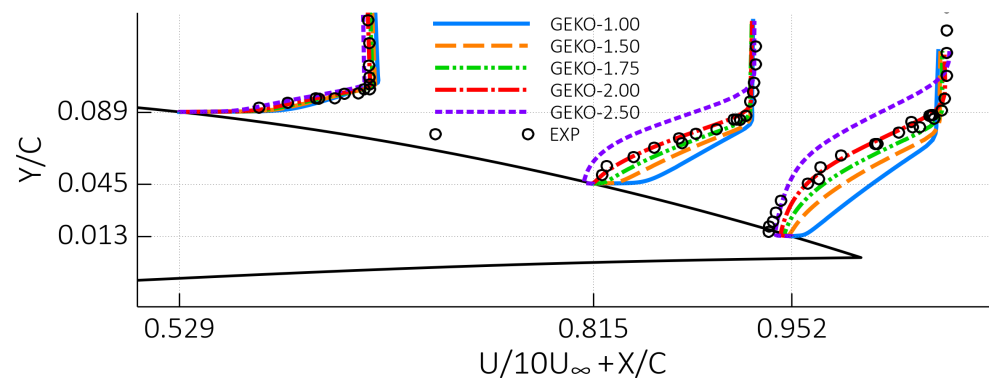


Courtesy A. Garbaruk



GEKO Model of Menter

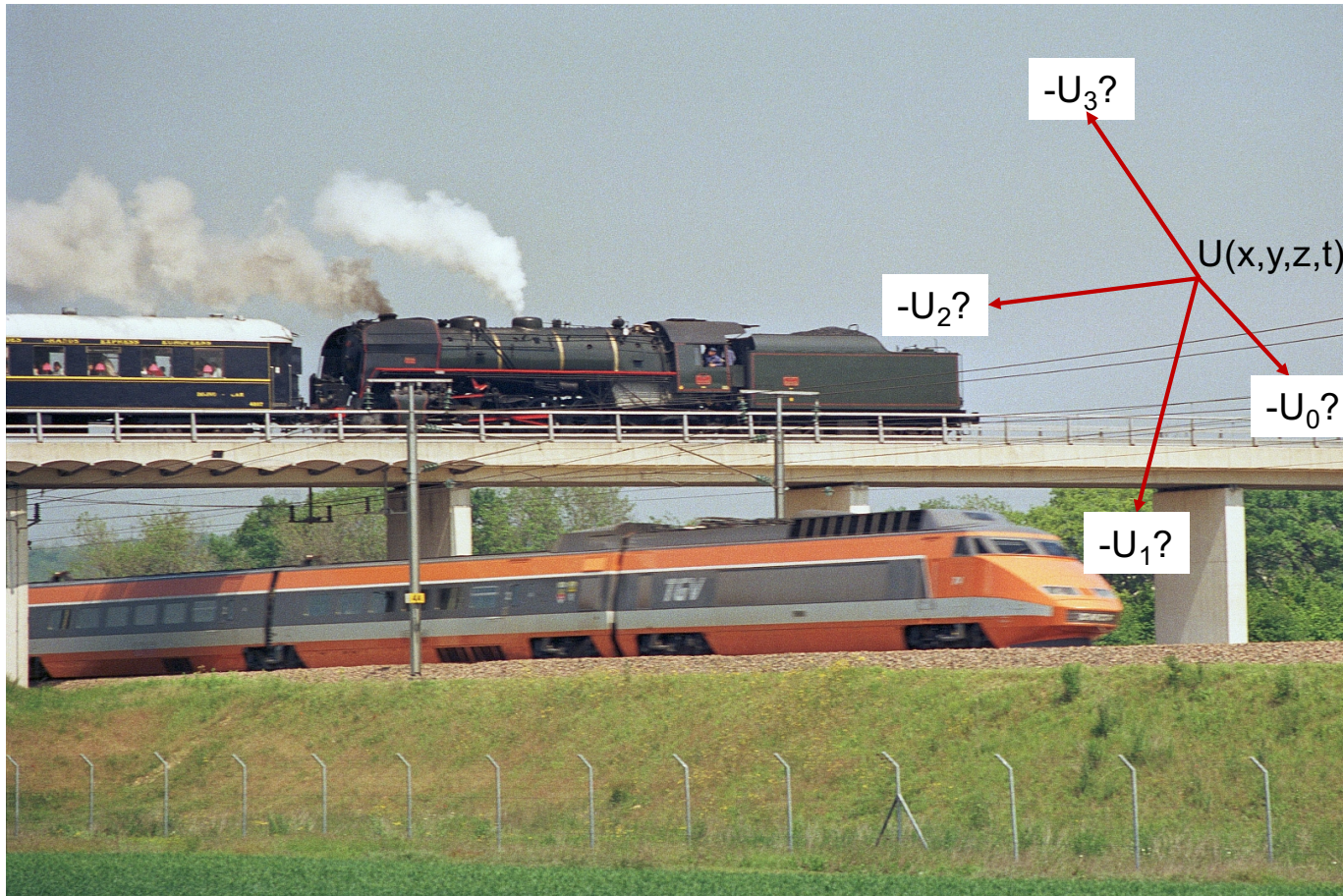
- GEneralized K-Omega
 - The formulas have not been published. It's coded only in ANSYS
 - However, the concept is clearly explained
- The user has SIX adjustable parameters, now “field variables”
 - Goal is to have at the user's discretion a single model that spans the behavior of many models, similar to “from k- ϵ to SST” but even wider
 - Each parameter controls a particular effect, e.g., separation, jet width, or corner vortices
 - They can take different values in different regions
 - A notable application is: thick wind-turbine airfoils
- The model is constrained to give the same flat-plate boundary layer
 - Boeing has secret versions of SA, which satisfy exactly the same constraint!



Hard Constraints on a Turbulence Model

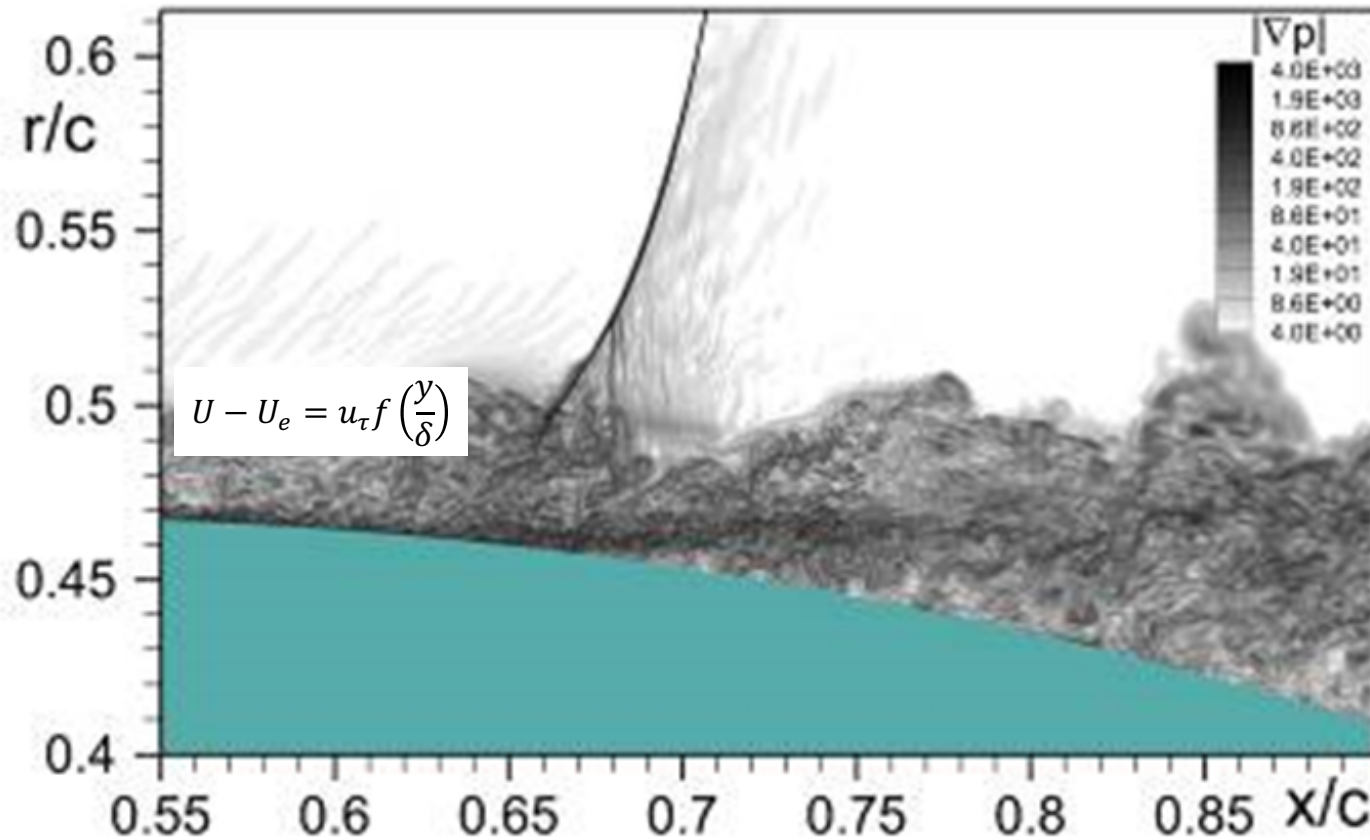
- Dimensional analysis
- Tensor symmetries
- Basic Hadamard well-posedness of the PDE
 - Unfortunately, hard to prove rigorously, but a failure is rapid in numerical solutions, even 1D
- Galilean invariance: exclude the velocity vector
 - And therefore “streamline curvature,” helicity ($\underline{U} \cdot \underline{\omega}$)...
- Exclude the acceleration: (and therefore the pressure gradient)
 - it can be different in flows that have exactly the same turbulence
- Exclude the molecular viscosity, outside the viscous/buffer layer
- Exclude numbers from the “flow problem,” such as U_∞ , τ_w , or Re_τ , or “reference length scale”
- Exclude axis-dependent measures, for instance the “Reynolds shear stress” versus “normal stresses”
- Exclude any dependence on the flow being steady
- Avoid a sensitivity to “ambient” values (those outside the turbulent layer)
 - Verify this by varying these values, with aggressive grid refinement
- Do not write a paper that gives 99% of the needed information (except for a vendor’s “secret sauces”)

Four Reference Frames

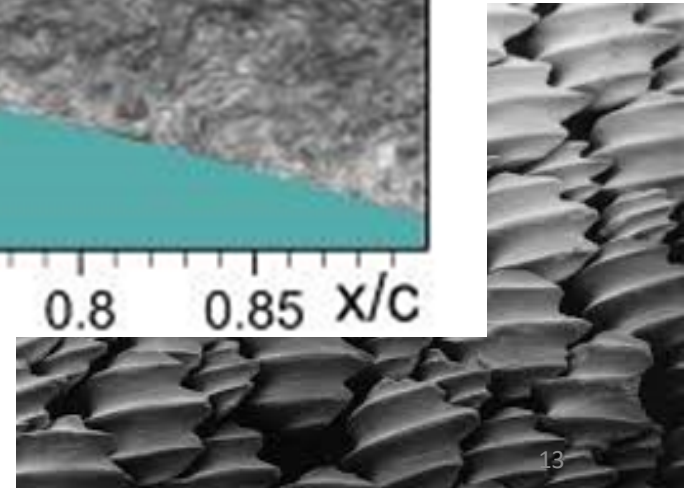
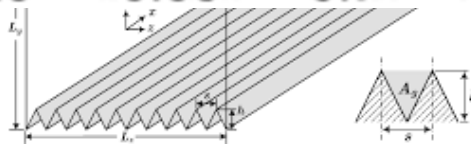
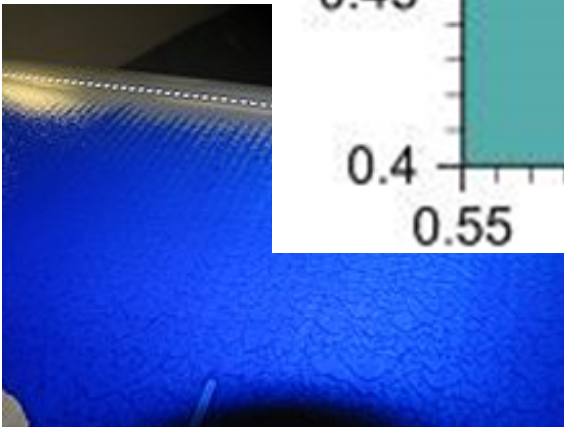


Excluding the Velocity: Possible Exceptions?

- Some
- The velocity can be
 - SA9
- However, it can be away

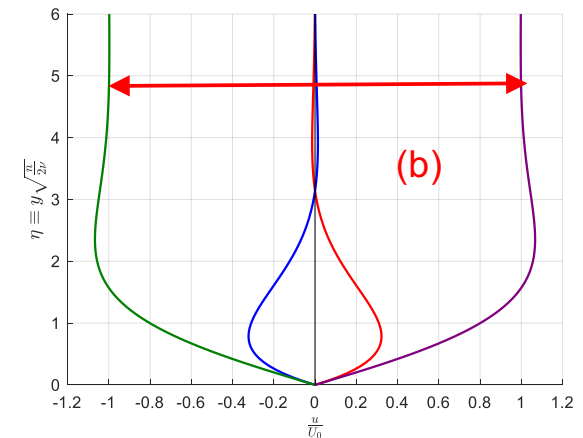
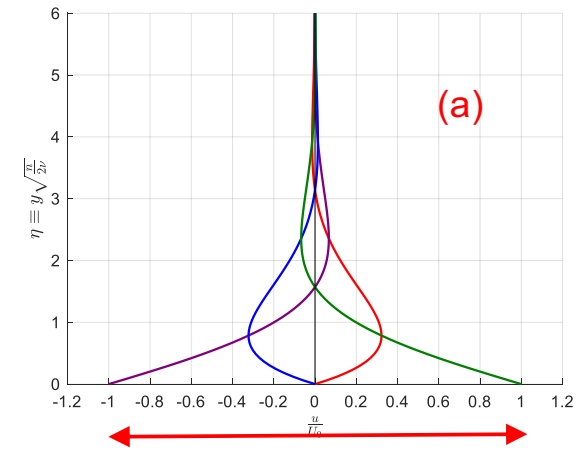


sition?
 II point
 IT to



Acceleration as Feature in a Turbulence Model

- I have been on this since 1999 (with Speziale)
- The pressure gradient is known to control transition, separation...
 - What better quantity to introduce?
- The issue is unsteady flows
 - Stokes' Second Problem:
 - Boundary layer with oscillating (air mass – wall) velocity difference
 $\Delta U = U_0 \cos(\omega t)$
- Use reference frame of air mass (a), or of the wall (b)
- The flows have different acceleration and pressure gradients
- They have exactly the same turbulence!
 - The same happens in channel flow: $W_{\text{wall}}(t)$ equivalent to $dP/dz(t)$
- In Real Life, consider vortex shedding, blade passing, etc.



Courtesy S. Sato 14

Strain and Vorticity as a Measure of Curvature Effect

- Many models use the strain magnitude S and the vorticity magnitude Ω
- In a purely azimuthal flow, with only $U_\theta(r)$, we have

$$r^* \equiv \frac{S}{\Omega} = \frac{\left| \frac{dU_\theta}{dr} - \frac{U_\theta}{r} \right|}{\left| \frac{dU_\theta}{dr} + \frac{U_\theta}{r} \right|}$$

- So that the ratio r^* is a measure of the rotation rate U_θ/r of the velocity vector for a particle, normalized by the shear rate “ dU_θ/dr ”
 - $r^* > 1$: concave curvature
 - r^* is properly invariant
- This is however not general at all. S varies for many other reasons
 - Also note that curvature corrections need large coefficients, $O(10) * (r^* - 1)$
- r^* is used in the SARC model, but not as primary sensor of “curvature”

Von Karman Length Scale

- Quite a few models use this length scale. In a simple $U(y)$ flow, it is:

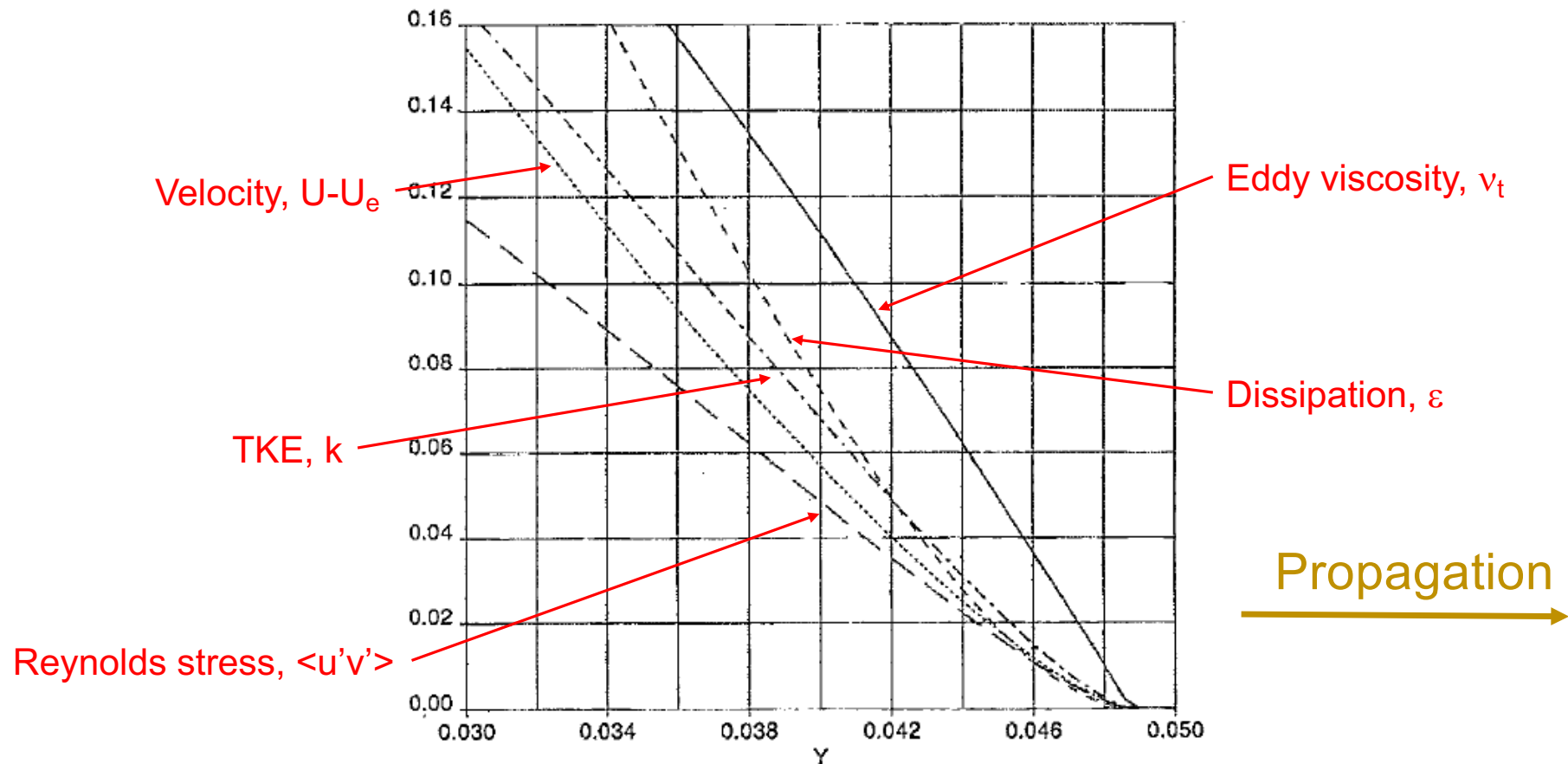
$$l_{VK} \equiv \frac{|\partial U / \partial y|}{|\partial^2 U / \partial y^2|}$$

- He meant it for the log layer: it returns y
 - Very tempting for “Wall-Distance Free” models!
 - It can be plausibly extended to 3D flow fields (but this is under challenge)
- At the turbulent-inviscid interface, $U(y)$ usually obeys a power law $U - U_e \propto (\delta - y)^p$
- Then, $l_{VK} = \delta - y$
- Usually, $1/l_{VK}$ is used, and this diverges like $1/(\delta - y)$
 - This magnifies destruction, which here is not countered by production
- l_{VK} is also 0 on channel centerline... and ∞ at an inflection point!
- WDF models (*improvements* on SA!) can easily fail to differentiate between the log layer and the other end of the inverted parabola
- This length scale is used in Menter’s SAS hybrid RANS-LES approach

Co-Existence of Turbulent and Non-Turbulent Regions

- External flows have turbulent layers surrounded by inviscid flow
- The value of the turbulence variables in the inviscid region should:
 - Be easy to set, with negligible influence on the turbulent region
 - Ideally, the variables can equal zero
 - Only exception: by-pass transition
 - Give eddy viscosity smaller than the molecular viscosity
 - Or, at least, give a high effective Reynolds number $U h / \nu_t$ even in small gaps
- The behavior of the equations at a laminar-turbulent interface is not trivial
 - Usually, the eddy viscosity has a jump in slope (weak solution of PDE)
 - Ramp solution is not difficult, up to two equations
 - The turbulent “ramp” must propagate into the inviscid region
 - This is the motivation for the k- ϵ /k- ω blending in the SST model
 - See Cazalbou, Spalart & Bradshaw 1994
 - Diffusion terms dominate

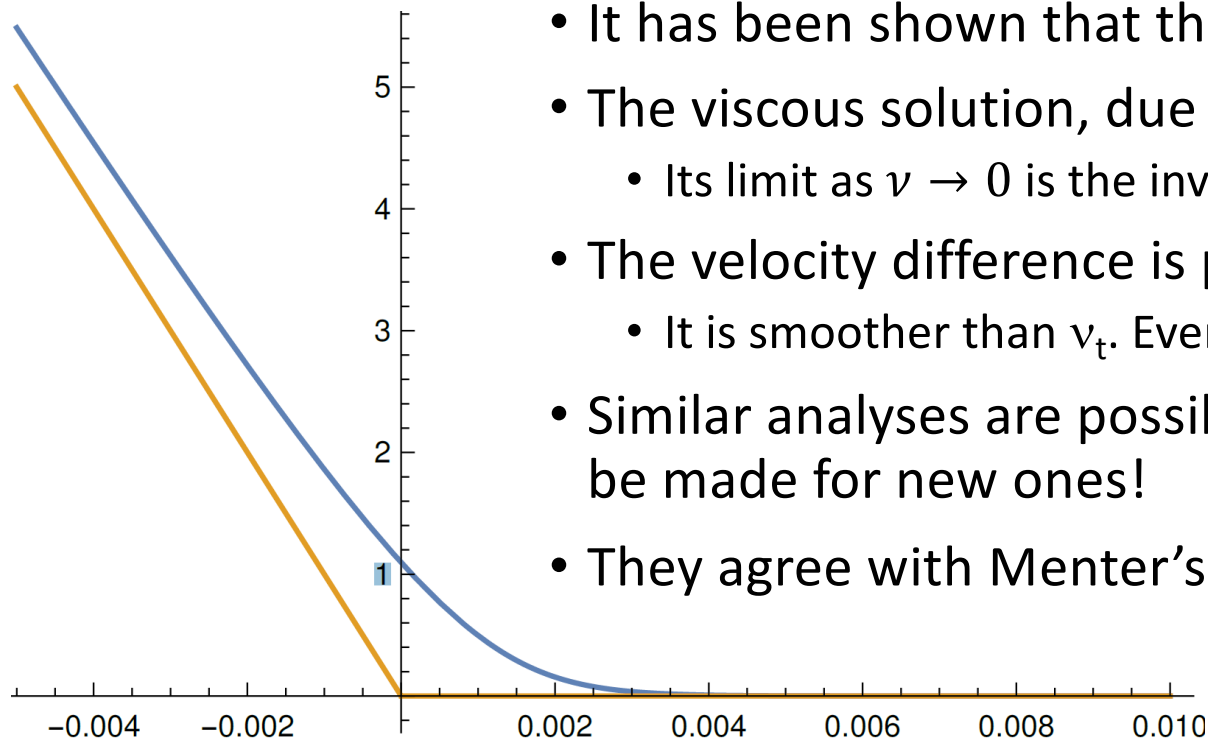
Turbulent Ramp of k- ϵ Model



Cazalbou, Spalart & Bradshaw 1994

Inviscid and Viscous Ramp Solutions for SA

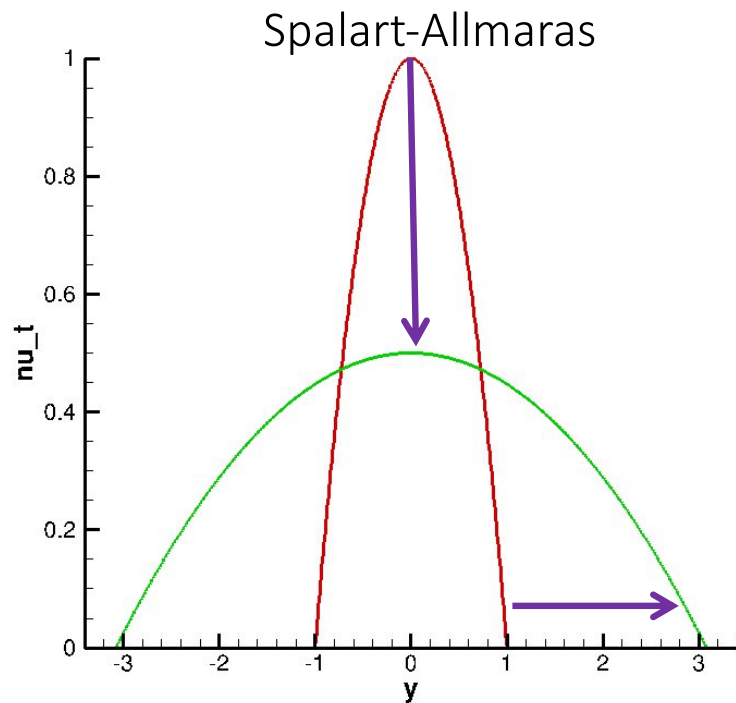
- Linear profile connecting to zero known since 1992
- This region is fully dominated by the diffusion terms
- The math is simple on either side of $y = 0$
- It has been shown that this is a weak solution at $y = 0$
- The viscous solution, due to S. Allmaras, is well-behaved
 - Its limit as $\nu \rightarrow 0$ is the inviscid weak solution
- The velocity difference is proportional to $|y|^{(1+cb2)/\sigma}$
 - It is smoother than v_t . Even $\partial U / \partial y$ is
- Similar analyses are possible for other models... and **should** be made for new ones!
- They agree with Menter's analysis of $k-\omega$



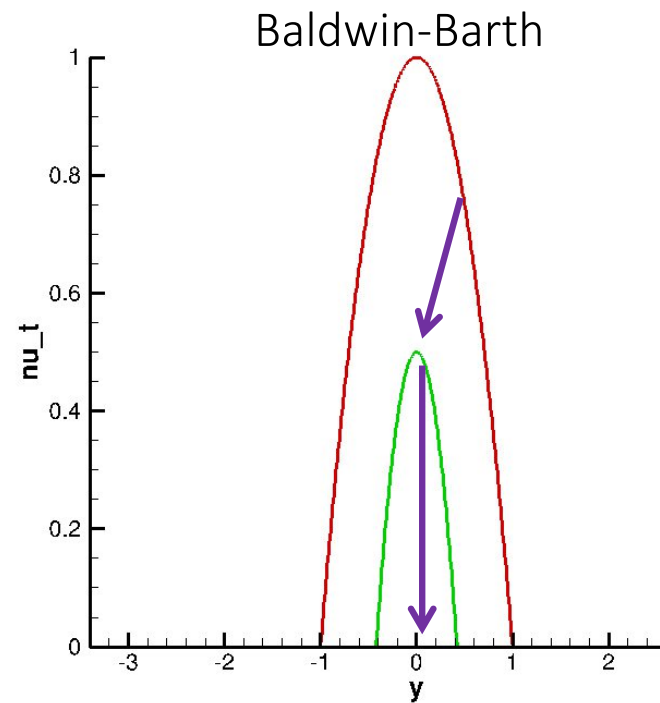
Parabola Solutions to Diffusion Equation

$$\frac{\partial \tilde{v}}{\partial t} = \frac{1}{\sigma} \left(\frac{\partial}{\partial y} \left(\tilde{v} \frac{\partial \tilde{v}}{\partial y} \right) + c_{b2} \left(\frac{\partial \tilde{v}}{\partial y} \right)^2 \right)$$

$$\tilde{v} = v_{tmax}(t) \max \left(1 - \frac{y^2}{\delta^2(t)}, 0 \right), \quad \frac{v_{tmax}}{v_0} = \left[1 + \frac{2v_0(3+2c_{b2})t}{\sigma\delta_0^2} \right]^{-1/(3+2c_{b2})}, \quad \frac{\delta}{\delta_0} = \left[1 + \frac{2v_0(3+2c_{b2})t}{\sigma\delta_0^2} \right]^{(1+c_{b2})/(3+2c_{b2})}.$$



Also works in
cylindrical and
spherical
coordinates!



Making a One-Equation Model, Starting From Two

- Many papers derive a one-equation model, say from k-ε. We have

$$\frac{Dk}{Dt} = \dots \qquad \frac{D\epsilon}{Dt} = \dots \qquad \frac{D}{Dt} \left(c_\mu \frac{k^2}{\epsilon} \right) = \dots$$

- To close system, add the Bradshaw Assumption: in shear flow,

$$v_t \frac{dU}{dy} = -\langle u'v' \rangle = a_1 k$$

- And of course, $\epsilon = c_\mu k^2 / v_t$. Most often, set $\sigma_k = \sigma_\epsilon$, and other simplifications
- An equation $Dv_t/Dt = \dots$ results, with the usual structure
- A destruction term is needed. Most often, it is a “ $c_{b2} < -1$ term” as in Baldwin-Barth; in other words, it’s dangerous!

Intermediate Constraints on a Turbulence Model

- Avoid the wall distance, d
- Avoid the wall-normal direction n_i
- Avoid high velocity derivatives
- Avoid singularities at the wall (e.g., $\omega = O(1/y^2)$)
- Guarantee realizability
- Do not use the absolute value of a legitimate “feature!” (e.g. Pope term, dP/dx , curvature...)
- Avoid breakdown when a quantity goes slightly negative
- Use only differentiable functions (no min or max or absolute value)
- Avoid model activating in irrotational regions (“stagnation-point anomaly”)
- Avoid the von Karman length scale, “ $|U_y| / |U_{yy}|$ ”
- Do not sustain turbulence in a mature vortex

Turbulence in a Mature Vortex?

in 1995, Spalart & Garbaruk 2018

self-similar (it works for mixing layer)



μ

the eddy viscosity

Soft Constraints on a Turbulence Model

- Control damage to numerical convergence
 - Iterations
 - Grid spacing
- Control complexity; it leads to bugs
- Control versions; too many, and they are an “effort sink” for the community
- “Pick your battles:” focus on Reynolds stresses that drive the mean flow
 - Remember turbulence is “weak” and powerless in regions of intense distortion
 - A point well made by JCR Hunt

The Way Forward

- Effective strategies, based on both Natural and Artificial Intelligence (ML)
- Accurate, relevant data from DNS and experiments
 - But models have a “Structural Conflict”
- Critical examination of machine-learning studies

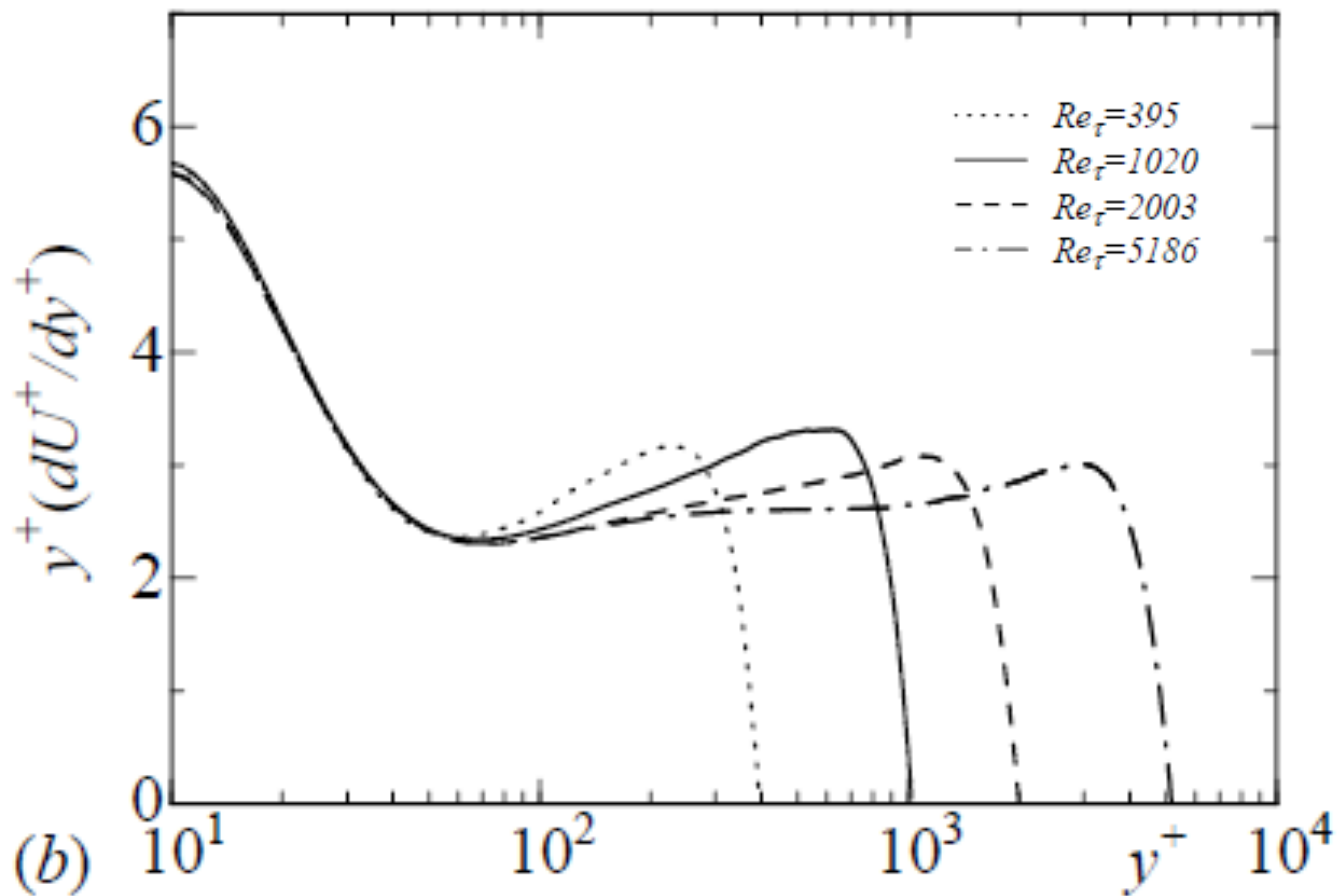
Effective Strategies

- Modeling still rests on “turbulence culture,” intuition, some math, and luck!
- The field has a high Barrier to Entry
 - Which requirements do you cover?
 - How do you make the model attractive?
 - How many flows do you test it on?
- Complete teamwork is our only chance
 - Recall “Stanford Olympics” (80’s) and Collaborative Testing of Turbulence Models (90’s)
 - Recent experiments, for the most part, have been designed in the open, CFD + modelers + experimentalists (esp. NASA Juncture Flow and “Bordeaux Bottle”)
 - The EU HiFi-TURB program combines DNS, Machine Learning and conventional modeling
 - Exceptions to full disclosure such as the GEKO and PowerFLOW (VLES) models are understandable

How does DNS Knowledge Enter RANS Models?

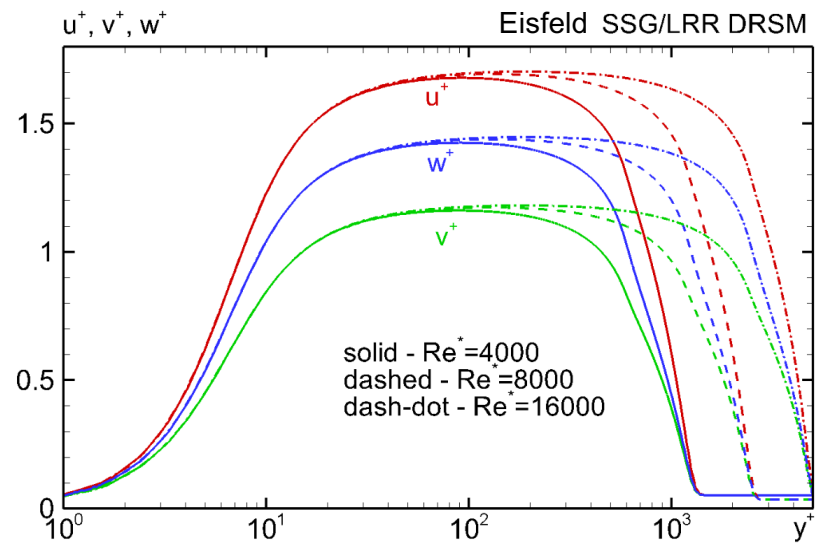
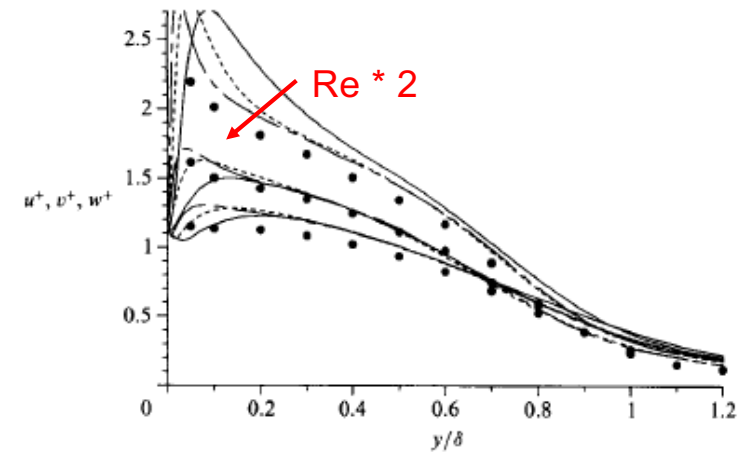
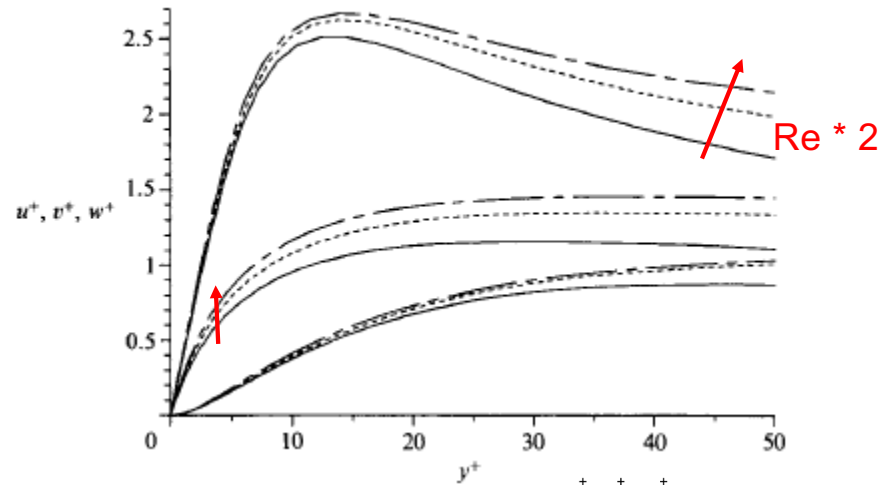
- This has been difficult. The benefit, so far, is almost invisible
- Big Data and Machine Learning could change this
- Channel flow is a good illustration:
 - The Reynolds number Re_τ has grown from 180 to 5200
 - Datasets from various groups form a “family,” to a good extent
 - However...
 - The velocity profiles do not show a well-defined log law
 - $dU^+/d(\log y^+)$ drifts
 - We cannot determine the Karman constant κ within $\pm 5\%$ (neither can modern experiments!)
 - Some of the Reynolds stresses behave against the “Law of the Wall”
 - They are not uniform in the log layer
 - At fixed y^+ , they depend on Re_τ
- RANS models of conventional type are *intrinsically unable* to capture these behaviors
 - “Conventional” meaning “driven by dU/dy in production terms” (for simple shear flows)
 - I call this the “Structural Conflict”
 - Some (Bradshaw, Saffman, Hunt, Durbin, Wilcox) have argued the conflict reflects “inactive” turbulence
 - With wavelength $\lambda \gg y$. Contributes to $\langle u'^2 \rangle$ but not $\langle u'v' \rangle$
 - If so, the models don’t need to include it
 - I don’t know of a quantitative definition
 - Machine Learning could blindly spend all its “capital” combating this conflict

“Karman measure” in Channel Flow DNS



Courtesy H. Abe

Real Life versus RANS Life/“Turbulence Theory”



Courtesy H. Abe
Rumsey data

Reflections on Machine Learning

- Avoid bombastic (“ronflant” in French) titles such as:
 - “Physics-Informed Machine Learning Approach for Augmenting Turbulence Models: A Comprehensive Framework”
 - 100 years ago, Prandtl and Taylor knew a lot about the physics
- Remember the “calling for universality”
- Remember a correction that is a function “ $\beta(x,y)$ ” in a single flow can be “instructive for a human modeller,” but it does not constitute a model
 - It is not clear how AI can choose which term to correct (e.g. production? destruction?)
- Writing that “a Neural Network was trained, and gave these results” does not give the reader a model
 - Notable exception: papers by Weatheritt & Sandberg: no NN, and specific PDE’s
 - W & S work with random mutations, between log and exp, and so on!
- The selection of the input quantities (“features”) is the core challenge
 - $\frac{\partial U_i}{\partial x_j}$; S_{ij} versus Ω_{ij} ; invariants and powers of $\frac{\partial U_i}{\partial x_j}$; d ; n_i ; $\frac{DS_{ij}}{Dt}$ (in RC and in Lag models); etc.
 - It is not clear how AI can do this
 - Except maybe down-select quantities that have impact out of the “human” list

Parting Thoughts

- The ultimate goal of “Natural or Artificial” Intelligence has been a single “best model money can buy,” as universal as can be
 - Proposing a model “good for one or a few flow modules” is only a short step
 - The GEKO “model suite” is a radical departure from this thinking
 - It has the rule that the flat-plate boundary layer cannot change, which we also apply to “SA-X”
- All models have a strong empirical (arbitrary) content in the “features” used
 - It is not clear AI can penetrate this part of the process
- Hard constraints when proposing a model include
 - Provide complete formulation and boundary conditions
 - Never use the velocity or acceleration (pressure gradient)
 - Model must be numerically stable inside the turbulent region
 - Understand the turbulent-inviscid interface:
 - Derive analytical solution connecting to negligible values
 - Test the model under vigorous grid refinement
- Softer requirements include:
 - Clarity, rationale, version control
 - Control of complexity
 - Be wary of the von-Karman length scale
 - Extinguish the eddy viscosity in a mature vortex